A Diagrammatic Approach to Teaching the Energy Conservation Law

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Abstract

A class of diagrams analogous to the free body diagram of mechanics is developed to describe energy transfers and transformations. These diagrams, if specifically linked to equation based descriptions of energy processes can only be drawn correctly if students accurately describe the energy changes and transfers taking place during a particular physical process. The advantage of drawing diagrams of this type is that students are asked to specifically address important issues related to the energy concept such as system boundaries, and the difference between transfer quantities and actual energy changes.

Introduction

There is a long history within physics, of using diagrams to understand physical processes. Often a direct connection is established between the diagram and a related mathematical formulation. In this sense both Feynman diagrams, used to illustrate and keep track of electromagnetic processes at the subatomic level, and free body diagrams, used in elementary mechanics to delineate forces acting on a body, are shaped from the same mold: when diagrams of this type of drawn there exists direct correspondence between the diagram and an equation or set of equations. In the case of the free body diagram one is visually representing the summation of forces that will enter into a formulation of Newton's second law of motion ($m\mathbf{a} = \mathbf{F}_{\mathbf{net}}$). Any experienced teacher of physics is aware of how misconceptions regarding forces and applications of Newton's second and third laws of motion in particular arise when students are asked to draw free body diagrams of even relatively simple interactions. For example, students often include fictitious forces in their diagrams (only some of which can be labeled as inertial forces) , and are unclear about the assignment of third law force pairs. Thus the drawing of free body diagrams with or without the accompanying equations can be a pedagogically significant exercise.

Why not approach the teaching of the energy concept in the same manner by asking students to draw diagrams based on specific equation based rules representing the energy transfers occurring during physical processes? Proper use of the mathematical formalism associated with the energy concept especially in applications beyond mechanics requires an ability to distinguish between different forms of energy and between energies (e.g. kinetic, thermal etc.) and energy transfers such as heat and work. As with the free body diagram it would offer the opportunity to address student misconceptions and clarify concepts. In addition, focusing on diagrams means there is less of a need to develop all of the mathematics associated with particular energy transformation and the full power of the energy conservation principle can be developed earlier in a course of study. It is the purpose of this paper to discuss the rules governing such diagrams and to offer examples that illustrate the utility of such an approach.

We note that the idea of drawing diagrams to represent energy changes or transfers is not in and of itself original and other authors have addressed the issue in journal articles and/or curriculum. In some sense we are refining the so-called energy-budget diagram (Licht et al 1990, van Huis and van den Berg 1993) which is also equation based. Diagrammatic approaches not directly tied to specific mathematical formulations such as the energy carrier concept (Schmid 1982) may be appropriate at lower grade levels when exploration is all that is desired, but our view is that if a formal concept of energy and its conservation is to be developed, careful use of language and/or mathematics to distinguish between quantities and to precisely describe relationships is important as soon as the student is capable of acceptance of such ideas. An adage among music teachers is "practice makes permanent" meaning that bad habits are hard to break. Concepts such as force and energy are complex yet unambiguously represented in mathematical relationships. Their utility and application reside within the context of these relationships and other representations, graphical or verbal, should respect the rules of this contextual environment, even during the first pass through the subject. Representations of energy transfers such as atomic energy level diagrams, or potential energy diagrams can also be effective visual tools, however, their scope is limited.

Energy Relationships and Corresponding Diagrams

What might be the properties of a diagrams used to describe energy transformation processes? Like the free body diagram which is based primarily on Newton's second law, a properly drawn energy diagram of a physical process should be consistent with an equation based description of the same process. Energy equations tend to have a common structure, for example:

$$\Delta K = W \qquad (1)$$

$$\Delta E = W + Q. \qquad (2)$$

Equation (1) is the work-kinetic energy theorem of mechanics and (2) is essentially the first law of thermodynamics or alternately a formal application of the conservation of energy principle . K is the kinetic energy, W the work, Q the heat and E in equation (2) includes both external and internal energies. Both (1) and (2) sum interaction/transfer terms on one side of the equation (heat and work) and a change in the system energy is tabulated on the other side of the equation. Care should be taken when discussing (2) as it is not a literal translation of the usual verbal statement of the energy conservation principle (e.g. Hewitt 2006). That is, heat and work are not "energies" and (2) suggests that energy can both be "created and destroyed" when work and/or hear transfers occur. The energy conservation principle in equation form is a statement about a sum of the ΔE's for a system large enough to be considered closed. As such, no transfer terms (Q's and W's) should appear. Of course there is a need to describe open systems where (2) will contain transfer terms, but the essential point remains that in order to properly understand energy relationships such as (1) and (2) a disnction between energies and transfer terms must be made.

The structure and/or form of both the interaction and energy quantities depend of course on the definition of the system under consideration and only interactions with external agents in the system's environment are described with transfer quantities. A similar situation occurs with Newton's second law as only external agents can exert forces on an object. Typically free body diagrams do not contain accelerations, only forces. Since heat and work are not energies in the same sense that the product of the mass and acceleration is not a force, energy diagrams should either exclude energies, if only interactions are to be represented, or clearly distinguish energy changes from interactions. In mulling over this point we must recognize an essential difference between energy and force equations. Energy equations are not formulated to determine the magnitude and direction of an instantaneous vector quantity as in the case of force equations, rather they are principally bookkeeping devices to tabulate changes in scalar quantities. In many everyday phenomena (see examples below) energy changes internal to the defined system can result in observable behavior of the system without external work or heat transfers. In addition, although system definition and distinctions between internal and external are important in force law formulations their consideration mostly affects the tabulation of forces. The form of the left hand side of equation (2) is especially dependent on system definition, and a formal application of the energy conservation law cannot be achieved without careful consideration of the various energies involved in physical processes. For these reasons we argue that both the energies and interaction (transfer) quantities must be included in energy diagrams.

Free body diagrams being vector diagrams represent force arrows in terms of the real world direction of the force. Work and heat are scalars and as such have no direction, however, they have algebraic signs and an energy diagram must take into account this feature. If we were to focus on (2) alone then W and Q would each be positive if they described energy flows into the system and negative if they describe transfers out of the system. In the case of work, into the system is generally described as "on" the system and out of as "by" the system. (2) differs from (1) not only in that it includes heat flows but also in regard to the interpretation of the work. Usually discussions focused on equation (1) describe positive work as work that increases kinetic energy and as such some external agent does work "on" the system. The problem occurs when negative work is done. The kinetic energy is lowered, that much is clear, but is work being done "on" the object's environment in the thermodynamic sense? The answer is no, and any discussion of energy no matter where it starts should be directed towards formulations consistent with (2). We follow this approach in the rules we use, listed below, in drawing energy diagrams.

In the examples discussed below we include the following quantities.
(1)W, Q and K: work, heat and kinetic energy defined in the conventional manner. Q and W defined in a manner consistent with thermodynamics.
(2) U_g: gravitational potential energy defined in the conventional manner but considered to be stored in a system consisting of both the object and the gravitational field.
(3) E_T: thermal energy consisting of (in general) potential and kinetic energy associated with the center of mass motion of atoms and/or molecules.
(4) U_c: chemical energy which is a form of potential energy at the atomic level resulting from arrangements of atoms within molecules.

Energies are either kinetic, potential or a combination of the two. That is, they either quantify motion directly (kinetic) or the ability to gain or lose kinetic energy as a result of a position or arrangement based interaction (potential). All energies can be classified using this minimalist scheme. For example, thermal energy is a combination of the two as is wave motion although electromagnetic waves present a conceptual dilemma. Chemical energy is potential energy. Describing energy as the ability to do work muddies the waters, in our opinion, as a transfer quantity is confused with an energy. We view the energy concept as largely meaningless without the energy conservation law from which it secures a contextural foothold.

In drawing energy diagrams we observe the following conventions.
(1) Systems and subsystems are represented by labeled rectangles.
(2) Interaction quantities such as work and heat are represented outside the rectangles by labeled arrows. Arrows pointing into a rectangle represents a transfer in (work on/heat in) and vice versa (work by/heat out).
(3) Changes in energies are represented by labeled arrows within rectangles. An upward arrow indicates a gain and vice versa.
(4) Although it can be instructive to represent open systems, systems are closed wherever possible so as to account for transfer quantities, between open subsystems, in terms of energy changes. That is, where possible, our diagrams conserve energy in the sense of the overall energy conservation law which we consider to be primary.

Examples of Energy Diagrams

In figure 1 we treat a subject commonly covered in a course dealing with mechanics, namely, the interaction of an object with the Earth's gravitational field. Figure 1a is really nothing more than a variation on the free body diagram concept as a falling object is acted on by the force of gravity and the agent is treated as external. This is consistent with (1), but also points out the limitations of this relationship. An alternate formulation where the object loses potential energy as it gains kinetic is beyond (1) and is properly described by (2) without transfer terms:

$$\Delta K + \Delta U_g = 0$$

This relationship is often developed using a relationship of the form $\Delta U = -W_f$ together with (1), suggesting that potential energy is lost by the field alone as work is done. But this cannot be correct as it is the mutual separation of earth and object or alternately the object's position in the field that determines the potential energy. The system cannot be the object alone as the potential energy is stored in the combined system consisting of the object and the field even if it is only the object that gains kinetic energy. A proper energy diagram should address this issue and in figure 1b the falling object motion is treated with no transfer arrows. This differs significantly from a free body diagram. In figure 1c an object is lifted at constant speed by a person. Potential energy is stored in the combined object + field system as chemical energy is converted into gravitational potential energy via a transfer in the form of work. An application of (2) to each subsystem is possible here, hence, the linked diagrams with a transfer arrow. This is not possible for the situation in figure 1b.

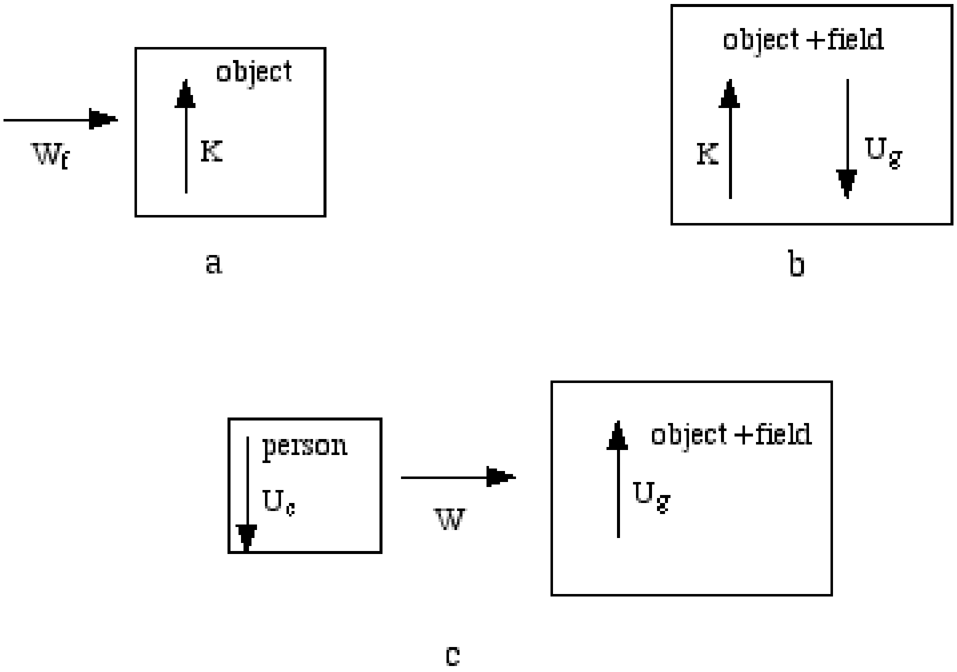

Figure 1: Interactions of an object with Earth's gravitational field. a) The field does work as an external agent on the object. b) The field and the object are seen as a single system. c) A person does work on the object+ field system while lifting the object converting chemical energy into gravitational potential energy.

Figure 1c corresponds to the following formulations of equation (2) for each subsystem

$$\Delta U_c = -W \tag{3a}$$

$$\Delta U_g = W \,. \tag{3b}$$

and the combined system of person+object+field conserves energy as can be seen by adding equations (3a) and (3b). An energy diagram using this system definition would include no transfer terms, similar to 1b, with chemical energy being converted to gravitational potential energy.

When frictional forces act similar decomposition issues occur. For example, in figure 2 we treat the common situation where a person pushes an object across a surface where sliding friction acts.

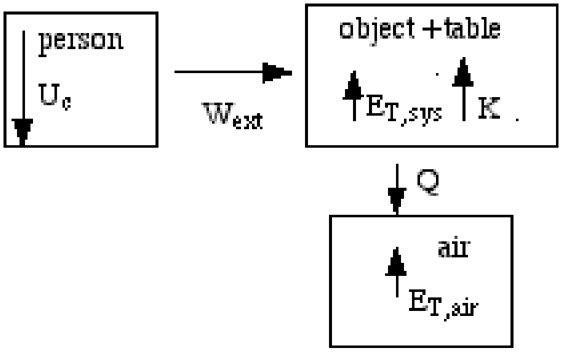

Figure 2. A person pushes an object across a table as sliding friction acts. Internal energy is developed in the object, the table and, as a result of a heat flow, the surrounding air.

As has been discussed by Erlichson (1977) and also Arons (1990) an application of (1) to this situation seems to imply that the sliding friction force does work equal to the product of the force and the center of mass displacement of the object. This quantity cannot be work in the thermodynamic sense as it is equal to the combined internal energy gained by the object, the surface and the environment. If this were the actual work done by the friction force its effect would be to divert energy exclusively out of the object. This not being the case, we can neither represent it as an outward arrow in a diagram nor treat it as work that lowers the energy of the object. The sliding friction force does work, but its calculation requires a more detailed analysis of the surface-object interface. Following Arons we resolve the dilemma by forming a combined system of surface and object for which the work done by the friction force, being internal, does not appear in a formulation of (2). The person on the other hand can be treated as a distinct subsystem external to the object +surface subsystem. Applying (2) to this linked system yields

$$\Delta K + \Delta E_{T,sys} = W_{ext} - Q \tag{4a}$$
$$\Delta E_{T,air} = Q \tag{4b}$$
$$\Delta E_{chem} = -W_{ext} \tag{4c}$$

Note that transfer arrows, unlike force vectors, are not attached to a particular subsystem alone in a closed system. The same arrow represents a loss by one subsystem and a corresponding gain in another. The sum of the equations (4a), (4b) and (4c) demonstrates the energy conservation law in that the right hand side sums to zero.

It is unlikely that equations 4a-c would be derived by most students especially in the context of a typical textbook exercise where the values of various numerical quantities are requested. But then the opportunity to apply the energy conservation law to a specific, very concrete, example is also lost. But imagine turning the exercise around by supplying either all or most of the numerical quantities and requesting an appropriate diagram be drawn given suggested system boundaries. For example,

A person does 50 J of work pushing a box across a table at constant speed. Within her body, 68 J of chemical energy are consumed and the surrounding air gains 18 J of thermal energy through a heat flow from the person's muscles. . The net gain of thermal energy by the box and the table *combined is 35 J as an additional increase of 15 J in the thermal energy of the air occurs through a heat flow from the box and the table. Draw an energy diagram for this process considering all three systems: the person, the box+table and the air.*

Even absent frictional effects there are many common systems, thought of as mechanical, where decomposition of the complex system into interacting subsystems is difficult or perhaps practically impossible. For example, a jogger who accelerates from rest to a running pace is acted on by a force of contact between his shoes and the ground which accelerates him. However, this force does no work and an energy description of the process is quite different from a force based dynamical description. Work is done within his body as muscles produce displacements and rotations but a dynamical description of these interactions would be very complex and in many cases unnecessary. Students may be led to believe that (1) is sufficient and that the contact force does work as it is responsible for the acceleration. Energetically the person can be viewed as a closed system, ignoring any heat flows out of his body, and the gain in kinetic energy results from a consumption of chemical energy as shown in figure 3. Even if heat flows or internal energy gains, are involved these are easily incorporated, the result being that not all of the chemical energy is converted to kinetic.

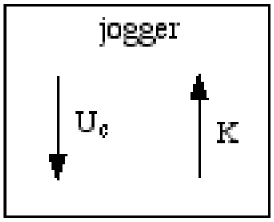

Figure 3. A jogger accelerating. All energy transformations are internal.

An application of (2) applied to this system yields: $\Delta K + \Delta U_c = 0$.

Discussion

The use of energy diagrams based on specific equations offers an alternate approach to teaching the energy concept, that emphasizes the differences between and the roles played by the various quantities that appear in energy relationships such as the conservation of energy law (2). We note that to follow this approach within a course on mechanics some effort must be devoted to developing student vocabulary so that terms such as heat and chemical energy have concrete meaning. However, one might also argue that the energy conservation law has no teeth without this expanded approach. By focusing on the often overlooked issues such as defining system boundaries, distinguishing transfers from energy changes and correctly mapping the flow from one subsystem to another students, even if all numerical values are given, are forced to apply the energy concept in its full form, which often is not the case if a standard equation based approach is followed.

In fact we recommend asking students to draw diagrams for situations where all or most numerical values are given, as issues rarely addressed directly by students such as system boundary definition can be highlighted. It is unlikely that especially in early treatments of the subject too much would be given away as students would still be struggling with concept formation. With time more autonomy, for example in regard to system definition, and less specific information should be provided. Research comparing expert and novice problem solvers (Larkin et al 1980) has shown that the relevance of devices such as free body diagrams is immediately obvious to the expert who views the equation(s) in a broader context than the novice. The expert dwells on conceptual issues such as system definition and coordinate system prior to formulation of equations, whereas the novice goes straight to the equation. A diagram based, equation free or equation minimized approach to the teaching of the energy concept might foster in students a greater appreciation of the context within which the equations derive meaning.

Our approach treats the energy conservation law as primary and is broader than restrictive relationships such as the work-kinetic energy theorem, (1) . As emphasized by Arons (1990) the energy conservation law cannot be derived from other relationships such as Newton's second law and energy diagrams such as figure 1a are really nothing more than modified free body diagrams. Applying the energy concept to mechanical or non-mechanical systems is for the most part vastly different than applying Newton's laws of motion to the same system. We should teach energy with that in mind, which dictates a broader approach that expands student vocabulary in a sufficient manner so as to close most systems when drawing diagrams and describing physical processes in general. Within this expanded approach students are able to correctly address situations where systems such as accelerating automobiles gain kinetic energy without work being done. It is only through addressing situations where force and energy descriptions appear to be in conflict that students can come to appreciate the broader context of the energy conservation law and the energy concept specifically.

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