Calculating the Rotational Velocity of Mercury from the Data

You will want to use the following formulas to carry out the steps in transforming your raw data into values of the rotation period of Mercury. Make sure you convert all measurements to the proper units as you go along.

Mercury’s rotational velocity, \( V \), is calculated from these geometrical relationships:
- \( R \) is the Planet’s radius
- \( d \) is the delay distance
- \( V_0 \) is the measured component of the rotational velocity parallel to the line of sight at the point.

1) Calculating \( d \), distance traveled by the delayed beam beyond the sub-radar point.

We simply use \( \text{distance} = \text{rate} \times \text{time} \), but since we are measuring an echo, which has to travel over the same path twice (down and back) we take half this value.

\[
d = \frac{(c \Delta t)}{2}
\]

Where \( c \) is the speed of light in meters per second \((3 \times 10^8 \text{ meters/sec})\) and \( \Delta t \) is the time delay for the particular pulse in seconds. (Note that one microsecond = 1 \( \mu \text{sec} = 10^{-6} \) seconds).

As an example, the delay distance \( d \) for the \( 120 \times 10^{-6} \) s delay echo pulse is calculated for you below:

\[
d = \frac{(c \Delta t)}{2} = \frac{(3 \times 10^8 \times 120 \times 10^{-6})}{2} = 18000 \text{ m}
\]
The Rotation of Mercury by Doppler Effect - Calculation

2) Calculating \( x \). This is the distance parallel to our line of sight from the center of Mercury to the point from which the echo comes back. It is just the radius of Mercury minus the distance \( d \) calculated in item 1, where the radius of Mercury is \( R_{\text{merc}} = 2.42 \times 10^6 \) meters.

\[
x = R_{\text{merc}} - d
\]

Note that since the echoes we measure come back from points not far back (only a few kilometers) from the sub-radar point, \( x \) will be only slightly smaller than \( R \).

As an example, the distance \( x \) of the echo pulse of delay time \( 120 \times 10^{-6} \) s is calculated for you below:

\[
x = R_{\text{merc}} - d = 2.42 \times 10^6 - 18000 = 2402000 = 2.402 \times 10^6 \text{ m}
\]

3) Calculating \( y \). This is the distance perpendicular to our line of sight to the extreme outer edge of the region of Mercury from which the echo comes back. It is found by noting that \( y \) is one side of a right triangle whose hypotenuse is the radius of Mercury, and whose other side is \( x \).

\[
y = (R_{\text{merc}}^2 - x^2)^{1/2}
\]

As an example, the \( y \) of the echoed pulse of delay time \( 120 \times 10^{-6} \) s is calculated for you below:

\[
y = (R_{\text{merc}}^2 - x^2)^{1/2} = ((2.42 \times 10^6)^2 - (2.402 \times 10^6)^2)^{1/2} = 294612 \text{ m}
\]

4) \( \Delta f_{\text{right}} \) and \( \Delta f_{\text{left}} \) are measured and recorded previously in the “Data Collection” section of this lab.

5) Calculate \( \Delta f_{\text{total}} \). This is the shift in frequency due to the Doppler Effect of the Mercury’s rotational velocity. You need simply note that one side of Mercury that is rotating toward you as fast as the other side is rotating away. So the difference in the frequency shifts from the two extremes edges, \( \Delta f_{\text{right}} \) and \( \Delta f_{\text{left}} \) is twice the shift due to rotational velocity.

\[
\Delta f_{\text{total}} = (\Delta f_{\text{right}} - \Delta f_{\text{left}}) / 2
\]

As an example, the \( \Delta f_{\text{total}} \) for the echoed pulse of delay time \( 120 \times 10^{-6} \) s is calculated below using the sample data from “Data Collection” section (note: your measurements may be different because we make observations on different days):

\[
\Delta f_{\text{total}} = (\Delta f_{\text{right}} - \Delta f_{\text{left}}) / 2 = (-82486.82 - (-82488.98)) = 1.08 \text{ Hz}
\]

6) Calculate \( \Delta f_{c} \). This is the shift in frequency corrected for the fact that this is an echo—the shift is twice that produced by a source which is simply emitting at a
The Rotation of Mercury by Doppler Effect - Calculation

Known frequency. This is because the pulse arriving at Mercury appears shifted as seen from the surface, and then it is shifted again because the surface of Mercury is moving as seen from the Earth.

\[ \Delta f_c = \Delta f_{\text{total}} / 2 \]

As an example, the corrected frequency of the echoed pulse of delay time \(120 \times 10^{-6}\) s is calculated below,

\[ \Delta f_c = \Delta f_{\text{total}} / 2 = 1.08 / 2 = 0.54 \text{ Hz} \]

7) Calculating \( V_o \). This is the component of the rotational velocity of the edge of Mercury along the line of sight at the point from which the echo returns. We simply apply the Doppler equation to the observed frequency shift.

\[ V_o = c(\Delta f_c / f) \]

Where \( c \) is the speed of light \((3 \times 10^8 \text{ m/s})\), and \( f \) is the frequency of the initial pulse recorded on top the DATA TABLE.

As an example, the \( V_o \) of the echoed pulse of delay time \(120 \times 10^{-6}\) s is calculated below, from step 6) we recorded that \( f = 430 \text{ MHz} = 430 \times 10^6 \text{ Hz} \), hence

\[ V_o = c(\Delta f_c / f) = (3 \times 10^8)(0.54/(430 \times 10^6)) = 0.37674 \text{ m/s} \]

8) Calculating \( V \). This is the equatorial rotational velocity of the planet Mercury, and is just your measured velocity times a geometrical factor \( R_{\text{merc}} / y \) which corrects for the fact that the velocity you measure is only the component of the rotational velocity directed along your line of sight, and that the component perpendicular to the line of sight produces no measurable Doppler shift.

\[ V = V_o \left( R_{\text{merc}} / y \right) \]

As an example, the \( V \) of the echoed pulse of delay time \(120 \times 10^{-6}\) s is calculated below,

\[ V = V_o \left( R_{\text{merc}} / y \right) = 0.37674(2.42 \times 10^6/294612) = 3.09 \text{ m/s} \]

9) Calculating \( P_{\text{rot}} \). For each of the delayed echoes you can now calculate a rotational period for the planet \( P_{\text{rot}} \) by dividing the circumference of Mercury \((2\pi R_{\text{merc}})\) by its velocity \( (V) \).

\[ P_{\text{rot}}(\text{seconds}) = 2\pi R_{\text{merc}} / V \]

As an example, the \( P_{\text{rot}} \) for the echoed pulse of delay time \(120 \times 10^{-6}\) seconds is calculated below:

\[ P_{\text{rot}}(\text{seconds}) = 2\pi R_{\text{merc}} / V = 2\pi \times 2.42 \times 10^6 / 3.09 = 4920812 \text{ seconds} \]
The Rotation of Mercury by Doppler Effect - Calculation

10) The $P_{rot}$ you calculated in the previous step is in the unit of seconds, we can convert it into days by dividing the number of seconds in a day (86,400) into it.

$$P_{rot\ (days)} = \frac{P_{rot\ (seconds)}}{86,400}$$

As an example, The $P_{rot}$ (in seconds) of the echoed pulse of delay time $120 \times 10^{-6}$ s is calculated below:

$$P_{rot\ (days)} = \frac{P_{rot\ (seconds)}}{86,400} = \frac{4920812}{86400} = 57\ days$$

11) Repeat calculations 1) through 10) for all four delay times, record your data into appropriate column in the DATA TABLE. You can check your results using the method described below. When all of your results are reasonable, you can move on to the “Question” section of this lab.

**Check your results**

Due to the complexity of the calculation, this program has a special feature to help you check your results as you go along the calculation. From the main window entitled “CLEA exercise- Mercury Rotation”, select **Work Sheet**, a “Mercury Lab Data Table” window will open. You can enter your results into appropriate boxes and click **Check** at the end of that column. A message will pop up to tell you if your answer is reasonable. Even though this feature does not calculate or correct answers for you, it is a great tool to help you find out where you might have make a mistake in the previous steps.